

Hand-on creative activities created using game based learning for high-school students during virtual lessons in mathematics

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-online teaching

-less extrinsic motivation, fewer grades

-revision needed

Game based learning and Gamification

- + Gamification is the use of game design elements in non-game contexts. [Deterding, et al., 2011]
- + Game-based learning refers to the borrowing of certain gaming principles and applying them to real-life settings to engage users. [Trybus 2015]

Assessment criteria

	0	1	2
Level of knowledge needed to solve the tasks	The tasks are mostly too easy and are not helpful for revision	The tasks allow only the medium educational outcomes to be revised.	The tasks allow also higher educational outcomes to be revised.
Correctness of the solutions of the tasks	Solutions of the tasks are mostly incorrect	The solutions of the tasks are mostly correct	The solutions of the tasks are completely correct and are given in the step-by-step shape.
Work organization	Some members of the group didn't partake in the project	The work load was not equally distributed and/or it is not fully clear who did which part.	The work load was equally distributed and it is fully clear who did which part.
Personal engagement	The project does not reach the standard described by the other descriptors.	There is evidence of some personal engagement.	There is evidence of a significant personal engagement.

Points	0-1	2-3	4-5	6	7-8
Grade	1	2	3	4	5

Example 1: PPT with tasks and links leading the user through an adventure,
K. Tunjić, 2.A

JUMANJI

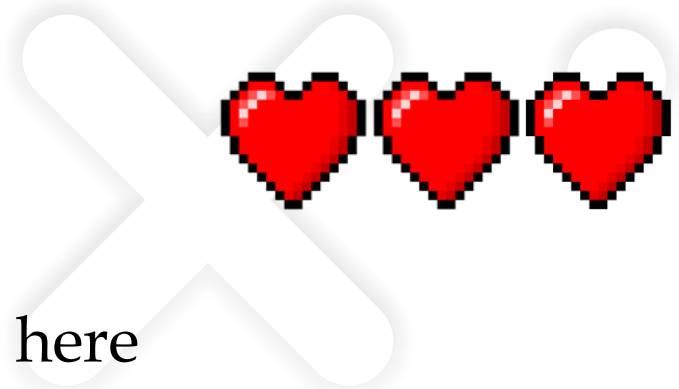



Welcome to Jumanji!

Jumanji is in great danger. It's up to you to lift the curse.
Use your math skills and retrieve the long-lost gem hidden in the pyramid.
In order to move through the game, do not use the keyboard,
but move by clicking the mouse on the answers and the arrow.
Remember! If you make a mistake you lose one life, you only have three,
each one is valuable.

The fate of Jumanji is in your hands!





Sphinx: Curious one, I see your courage,
answer correctly and you can drive out of here
in a carriage!

(Sfinga: Znatíželjničē, vidim hrabar si,
odgovori mi točan daj i život spasi.)

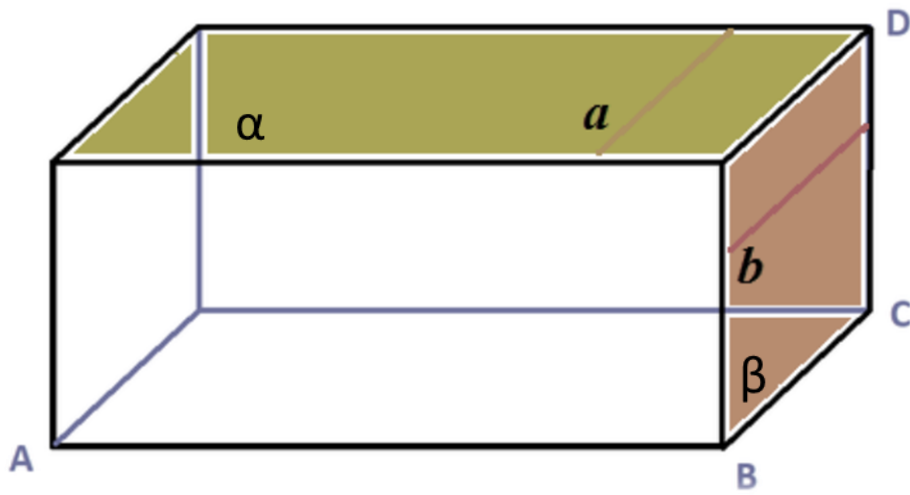
Line a lies in the plane α , line b lies in the plane β ,
 $a \parallel b$. Are you sure that: $\alpha \parallel \beta$?

No

Yes



Sphinx: „The answer you gave was correct,
to the entrance of the pyramid you can connect!”
(Sfinga: „Točan odgovor to je znaj,
ući u piramidu smiješ
zato što si dao odgovor taj!”)





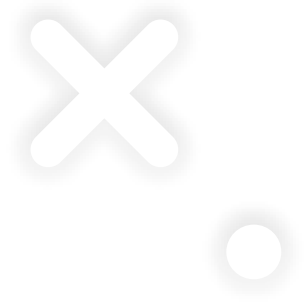
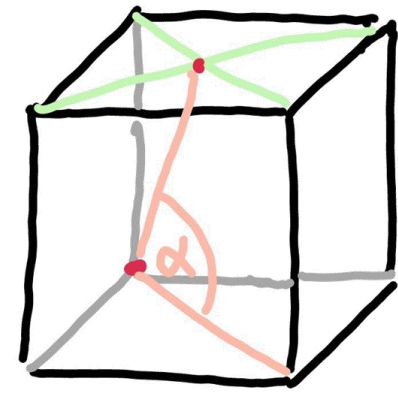
As soon as you entered, you were greeted by a precipice. On the floor next to you are a bow and string with an arrow, and on the ceiling of the corridor there is a place where the arrow could be well secured. The corridor is shaped like a cube. You are standing in one of its vertices, and the place where the arrow can be fixed is at the intersection of the diagonals of the ceiling. At what angle relative to the floor do you have to shoot the arrow?

30° 00'

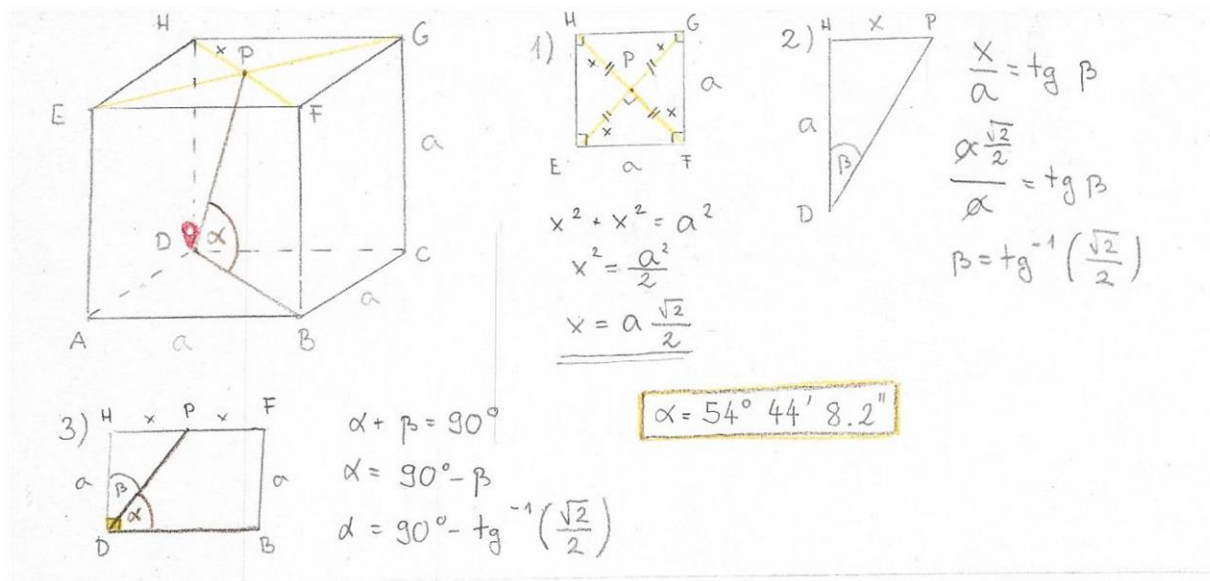
35° 16'

54° 44'

45° 00'



You shot an arrow. You are jumping over the precipice. Oh, no! The rope is not holding tight enough, your calculation was wrong. You fell into the abyss.



Fortune follows the brave! That's how you re-created yourself at the other end of the chasm.



Example 2: Snakes and Ladders – M. Pavlović, R. Domović, K. Miličić Juhas, Z. Mihaljević, B. Lešić, 2.A



4. Kocki volumena $\frac{1}{\pi}$ litre upisana je kugla. Odredi volumen koji pripada samo kocki.

3. Oko stošca volumena $144\pi \text{ cm}^3$ i visine 9 cm opisana je kugla polumjera 5 cm. Kolika je udaljenost od središta kugle do osnovke stošca?

2. Koliki je obujam kugle opisane pravilnoj trostranoj prizmi, ako je visina prizme 4 cm, a osnovni brid 6 cm

1. Polumjer presjeka kugle ravninom tri puta je manji od polumjera kugle. Ako je ravnina od središta kugle udaljena 4 cm, kolika je površina presjeka?

8. Oplošje pravilne četverostrane piramide kojem su pobočke jednakostranični trokuti je 108 cm^2 , a kut između pobočke i osnovke je 60 stupnjeva. Koliki je volumen te piramide

7. Piramida visine 24 cm i površine baze 100 cm^2 presječena je ravninom koja je paralelna s bazom i od vrha je udaljena 8 cm. Koliki je volumen piramide

6. Unutar kugle radijusa 13 cm nalaze se dvije paralelne ravnine. Jedna je od središta udaljena 5, a druga 12 cm. Koliki je volumen nastalog sloja?

5. Kugla opisana kocki ima oplošje $108\pi \text{ cm}^2$. Koliki je volumen kocke?

12. Što je krnja piramida?

11. Kada je piramida uspravna?

10. Pravilna četverostrana piramida je presiječena ravninom tako da je presijek jednakostranični trokut opsega 24 cm. Koliki je volumen piramide

9. Bazi pravilne šesterostrane piramide je opisana kružnica promjera 12 cm. Koliki je volumen piramide ako je duljina bočnog brida 10 cm.

16. U posudu oblika kvadra, stranica $a = 10 \text{ cm}$, $b = 15 \text{ cm}$ i $c = 17 \text{ cm}$, nalili smo vodu do dvije trećine visine (stranice c) posude. U vodu je ubačena kocka. Ukupan volumen uronjene kocke i vode je 1825 cm^3 . Koliko iznosi oplošje ubačene kocke?

15. Što su paralelopipedi?

14. Kako se izračunava broj bridova n-terostrane prizme?

13. Što je prizma?

Rješio Karlo Miličić Juhas(+13,14,15):

16.) U posudu oblika kvadra, stranica $a = 10$ cm, $b = 15$ cm i $c = 17$ cm, nalili smo vodu do dvije trećine visine (stranice c) posude. U vodu je ubačena kocka. Ukupan volumen uronjene kocke i vode je 1825 cm³. Koliko iznosi oplošje ubačene kocke?

4. $a = 10$ cm
 $b = 15$ cm
 $c = 17$ cm
 $V_{\text{uk}} = 1825$ cm³

$$V_{\text{uk}} = V_{\text{kocka}} + V_{\text{voda}}$$
$$V_{\text{kocka}} = V_{\text{uk}} - \frac{2}{3} a \cdot b \cdot c$$
$$V_{\text{kocka}} = 1825 - \frac{2}{3} \cdot 10 \cdot 15 \cdot 17$$
$$V_{\text{kocka}} = 1825 - 1700$$
$$V_{\text{kocka}} = 125$$
 cm³
$$V_{\text{kocka}} = a^3 \qquad O = 6a^2$$
$$125 = a^3 \qquad O = 6 \cdot 5^2$$
$$a = \sqrt[3]{125} \qquad O = 6 \cdot 25$$
$$a = 5$$
 cm $O = 150$ cm²

17.) Površina strana kvadra su u omjeru 2 : 3 : 5. Njegovo je oplošje jednako 300 cm². Koliki je obujam kvadra?

5. $a \cdot b : a \cdot c : b \cdot c = 2 : 3 : 5$
 $O = 300$ cm²

$$O = 2(a \cdot b + a \cdot c + b \cdot c)$$
$$300 = 2(2k + 3k + 5k)$$
$$150 = 10k$$
$$k = 15$$
 cm
$$a \cdot b = 30$$
 cm² $a \cdot b \cdot a \cdot c \cdot b \cdot c =$
$$a \cdot a \cdot b \cdot b \cdot c \cdot c = (a \cdot b \cdot c)^2 = V^2$$
$$b \cdot c = 25$$
 cm²
$$V^2 = 30 \cdot 45 \cdot 75$$
$$V^2 = 101250$$
$$V = \sqrt{101250} = 225 \cdot \sqrt{2}$$
$$V = 225 \sqrt{2}$$
 cm³

Example 4: Who wants to be a millionaire? – Trigonometry quiz –
J. Čorak, F. Kekez, 4.C



Question for 100 kuna

How many basic trigonometric functions are there?

A: one

B: four

C: two

D: three

Question for 1 million kuna



Which solution of the equation $\sin(x - \pi)\sin(x + 2\pi) = 3 \cos(x + 3\pi)\cos(x - 4\pi)$ belongs to the interval $[\pi/2, \pi]$?

A: $\pi/2$

B: $\pi/3$

C: $3\pi/4$

D: $2\pi/3$

Example 5: Trivial Pursuit game with tasks in various fields in Mathematics - A. Kučina, K.J. Kardum, A. Cvijanović, 4.c

Trivial pursuit

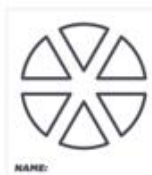


QUESTIONS	ANSWERS

QUESTIONS	ANSWERS



NAME:



NAME:

1. **EKSPONENCIJALNA FUNKCIJA**
2. **LOGARITAMSKA FUNKCIJA**
3. **STEREOMETRIJA**
4. **TRIGONOMETRIJSKE JEDNADŽBE I NEJEDNADŽBE**
5. **NIZOVI**
6. **GEMOTERIJA**

4. Odredi sliku funkcije. $(-2, +\infty)$

$$f(x) = 8^{-4x + \frac{3}{2}} - 2$$

5. Izračunaj $(x=4)$

$$2^{2x} * 3^{2x} = \frac{1}{6}(6^{3+x})^2$$

6. Izračunaj $(x = -3/2)$

$$0.5^{2x-1} + 0.25^{x-1} = 48$$

7. Izračunaj $(x_1=1/2, x_2=3/4)$

$$4^{4x} - 12 * 4^{2x} + 32 = 0$$

8. Izračunaj $(x=0)$

$$4^{2x+2} + 5 * 4^{x+2} = 6 * 4^2$$

9. Izračunaj $(x_1=1, x_2 = \log_6 3)$

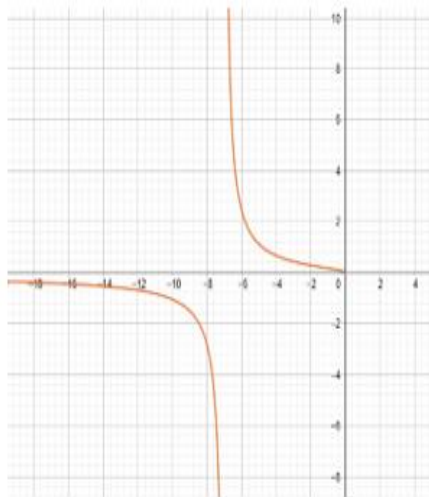
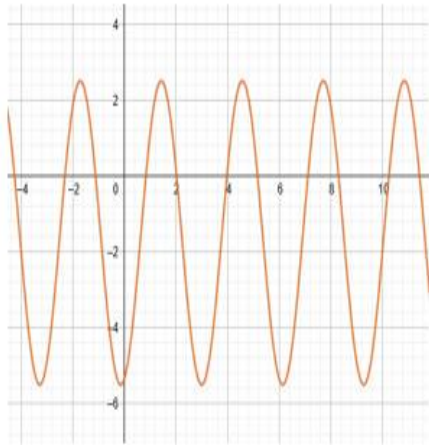
$$36^x = 3^{x+2} * 2^x - 18$$

10. Izračunaj $(x_1=1, x_2 = 0)$

$$2^x + 2^{1-x} = 3$$

11. Izračunaj $(x=1)$

Example 6: Memory game – the pairs of cards are not the same, but two functions on the pair of cards have something in common – M. Šola, 4.c



$f(x) = \log(2^{x-3} + 5)$	$f(x) = x^2 - 6x + 8 $
$f(x) = \frac{7\cos x}{3\sqrt{x}}$	$f(x) = \log_{\frac{1}{4}} \sqrt{\frac{x-6}{x^2}} + 2$
$f(x) = \log(17-x) + 6$	$f(x) = \frac{6x-13}{\log_3(x-10)}$
$f(x) = \sqrt{\ln(e^{2x} + 10)}$	$f(x) = \left(\frac{x+3}{x}\right)^x$
$f(x) = \frac{13x - \frac{5}{x}}{4x}$	$f(x) = \frac{5}{x^2} + \frac{13}{5x}$
$f(x) = \frac{20x^3 + 2x^2}{5x^4}$	$f(x) = 5x \operatorname{tg} x$

$f^{-1}(x) = \log_2(10^x - 5) + 3$	$\operatorname{Im} f = [-7, 0]$
$f'(x) = \frac{-2x + 10}{\ln 5 \cdot (-x^2 + 10x - 8)}$	$D = \langle -\infty, 2 \rangle \cup \langle \frac{3}{2}, +\infty \rangle$
$f^{-1}(x) = -10^{x-6} + 17$	$f'(x) = -\frac{13x + 50}{5x^3}$
$D = [-34, +\infty) \setminus \{7\}$	$f^{-1}(x) = \frac{1}{2} \ln(e^{x^2} - 10)$
$f^{-1}(x) = -\log_{\frac{3}{2}}(x + 8) + 5$	$\operatorname{Im} f = \langle 1, +\infty \rangle \setminus \{20\}$
$D = [-3, 2)$	$f'(x) = \frac{5}{2x^3}$

Examples of the solutions to Memory game

$$1. f(x) = \log(2^{x-3} + 5) \rightarrow f^{-1}(x) = \log_2(10^x - 5) + 3$$

$$2. f(x) = \log \sqrt{\frac{10x}{3x^2+7x}} \rightarrow f^{-1}(x) = \frac{10-7 \cdot 10^{2x}}{3 \cdot 10^{2x}}$$

$$3. f(x) = \sqrt{\frac{3x-8}{16x-9}} \rightarrow \text{Im}f = \mathbb{R} \setminus \left\{ \frac{\sqrt{3}}{4} \right\}$$

$$4. f(x) = \frac{20x^3+2x^2}{5x^4} \rightarrow f'(x) = -\frac{20x+4}{5x^3}$$

$$5. f(x) = \left(\frac{3}{2}\right)^{5-x} - 8 \rightarrow f^{-1}(x) = -\log_{\frac{3}{2}}(x+8) + 5$$

$$6. f(x) = \frac{1}{12+2^{-x}} \rightarrow f'(x) = \frac{\ln 2}{2^{x\left(\frac{1}{2^x}\right)^2}}$$

Example 7: „Choose-your-own story” depending on the answers to math questions – L. Mrvelj, 4.c

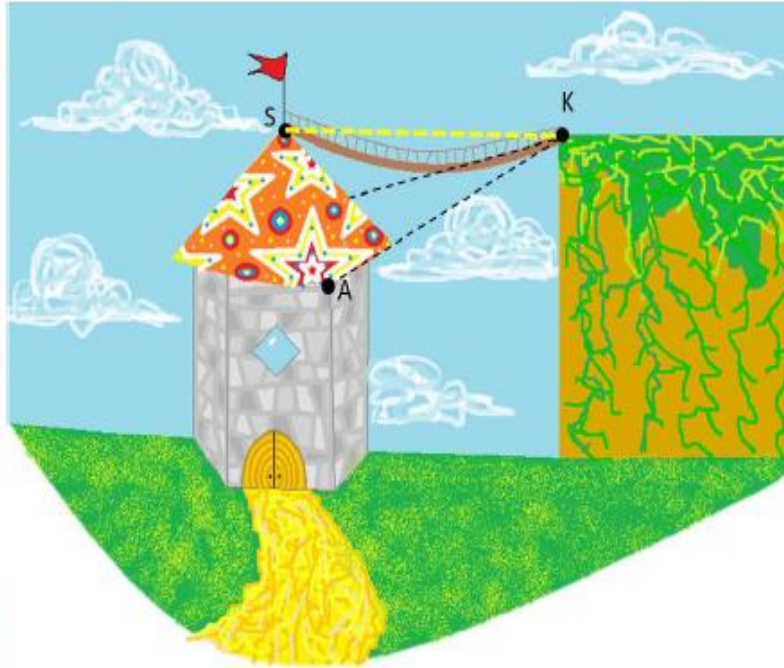
KRUŽNICA I PRIMJENA POUČAKA O SINUSIMA / KOSINUSIMA

Šefljislava i Žemljimir uspjeli su otvoriti vrata i ući u kulu! Prateći još jedne, sitnije stepenice, našli su se na samome vrhu kule, a pošto je krov kule imao oblik kruga, oni su prema tome bili u njegovu središtu. No, kuda ići sada?

Osvrnuli su se oko sebe i primjetili mostić koji je središte krova povezivao s obližnjim brdašcem u točki K. Mostić podržavaju dva konopa rastegnuta od točke K do dvaju točaka na krovu (A i B) te konopci funkcioniraju kao tangente kružnici krova.

Ako je kut $\sphericalangle ASB$ veličine 132° , kut $\sphericalangle AKB$ veličine 40° , a promjer kružnice krova iznosi 3m, kolika je zračna udaljenost vrha krova od točke K (duljina dužine \overline{SK})?

[Hint: Skica tlocrta uvijek pomaže!]



Šefljislava and Žemljimir managed to open the door and enter the tower! Following another, smaller staircase, they found themselves at the very top of the tower, and since the roof of the tower had the shape of a circle, they were therefore in its center. But where to go now? They looked around and noticed a bridge that connected the center of the roof to a nearby hill at point K. The bridge is supported by two ropes stretched from point K to two points on the roof (A and B), and the ropes function as tangents to the circle of the roof. If the angle $\sphericalangle ASB$ is 132° , the angle $\sphericalangle AKB$ is 40° , and the diameter of the roof circle is 3m, what is the aerial distance of the top of the roof from point K (length of the segment SK)? [Hint: A floor plan sketch always helps!]

8,75m (str.26) ↔ 9,52m (str.24) ↔ 10,16m (str.22)

Conclusion

+Cons:

- It is very hard to make sure all the members of the group work equally diligently
- The group working on a certain lesson revise that lesson very well, but lack practice in other fields

Conclusion

+Pros:

- Students were satisfied with the project and felt the process of making a game helped them learn a lot
- When playing other groups' games, they reported more engagement compared to the standard classroom practices
- Students reported the feeling of learning more by creating a game themselves, then by playing other group's game
- Most groups engaged deeply with the task and the grades were high (mostly 4 and 5)



Thank you for your attention

